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Otsu Algorithm
Optimal Global Thresholding

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Outline

- Introduction
- Class Separability Measure
- Threshold Selection Algorithm

Introduction

- About Global Thresholding
  - Thresholding: assign a binary value ∈ {0,1} to each image pixel $f(x,y)$ according to threshold t^*

$$g(x,y)=\begin{cases}0 & if \quad f(x,y)\le t^* \\ 1 & if \quad f(x,y)>t^*\end{cases}$$

  - Global Thresholding: t^* is a constant applicable over an entire image.

Introduction

- About Global Thresholding
  - Example: An 5x4 image with 8 gray levels (3 bits)

$f(.)$

2	2	2	1	4
2	1	6	4	1
2	2	1	6	5
2	2	5	5	5

$t^*=3$ →

$g(.)$

0	0	0	0	1
0	0	1	1	0
0	0	0	1	1
0	0	1	1	1

Global Thresholding → Threshold Selection

Introduction

- Formulation (Knowns)
  - let pixels of an image be presented in L gray levels $\{0, 1, \dots, L-1\}$
  - define normalized histogram of an image as $\{p_0, p_1, \dots, p_{L-1}\}$

$$p_i = n_i / N$$

    - n_i: number of pixels at gray level i
    - N: total number of pixels

Introduction

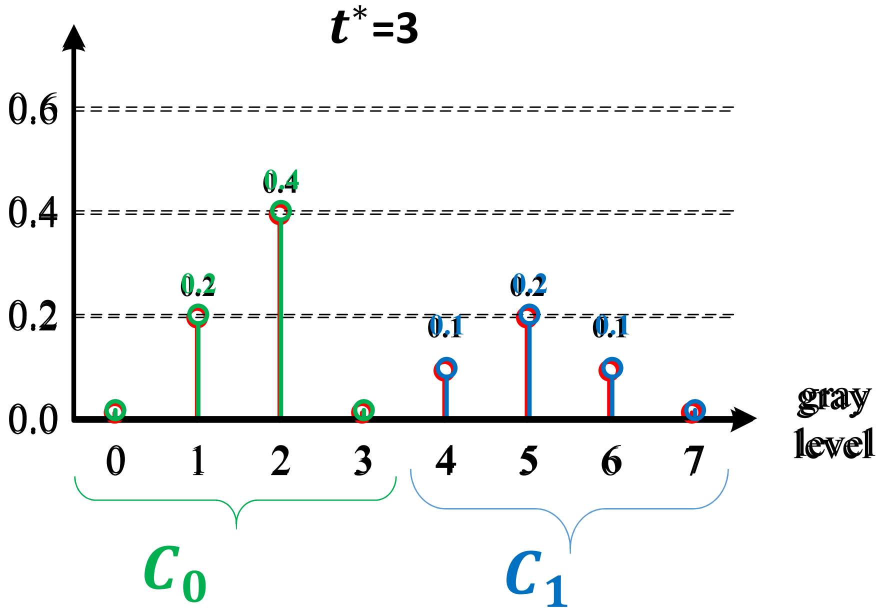
- Formulation (Objective)
  - find an optimal threshold t^* subject to a class separability measure
  - use t^* to dichotomize pixels into two classes
    - $C_0(t^*)$: pixels with levels $\{0,1,\dots,t^*\}$
    - $C_1(\mathrm{t}^*)$: pixels with levels $\{t^*+1,\dots,L-1\}$

Introduction

- Formulation

2	2	2	1	4
2	1	6	4	1
2	2	1	6	5
2	2	5	5	5

$$N = 20$$



$$p_0 = \frac{0}{20} = 0.0$$

$$p_1 = \frac{4}{20} = 0.2$$

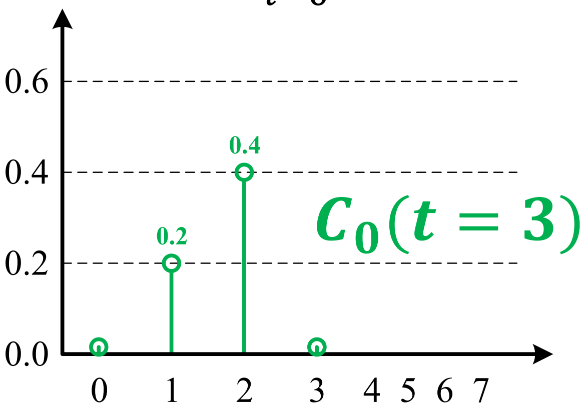
$$\vdots$$

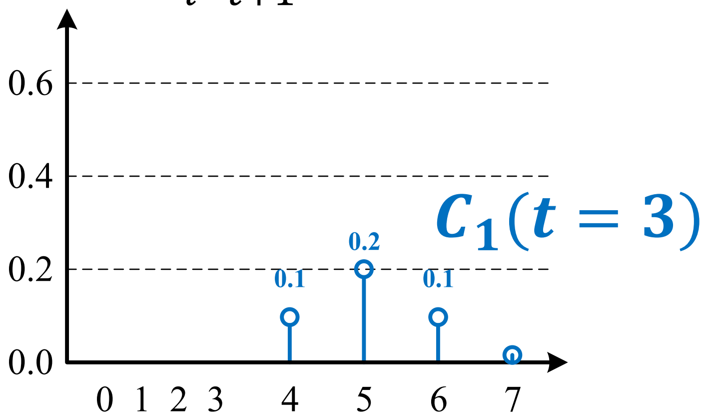
$$p_7 = \frac{0}{20} = 0.0$$

Class Separability Measure

- Statistical Terms: **Class Occurrence Probability**

$$P_0(t) = \sum_{i=0}^{t} p_i \qquad P_1(t) = \sum_{i=t+1}^{L-1} p_i$$





$$P_0(t=3) = \sum_{i=0}^{3} p_i = 0.0 + 0.2 + 0.4 + 0.0 = 0.6$$

$$P_1(t=3) = \sum_{i=4}^{7} p_i = 0.1 + 0.2 + 0.1 + 0.0 = 0.4$$

Class Separability Measure

- Statistical Terms

Class Mean

$Q_0(t)$

$$\mu_0(t) = \frac{1}{P_0(t)} \sum_{i=0}^{t} i \times p_i$$

$Q_1(t)$

$$\mu_1(t) = \frac{1}{P_1(t)} \sum_{i=t+1}^{L-1} i \times p_i$$

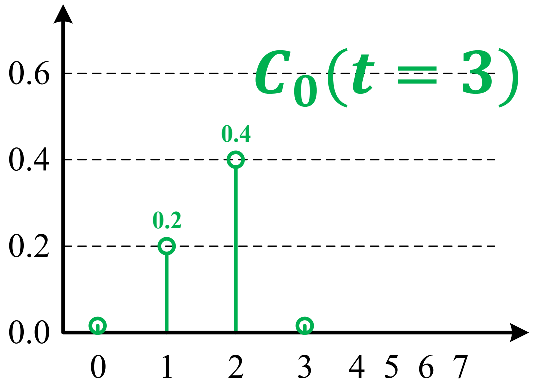
Class Variance

$$\sigma_0^2(t) = \frac{1}{P_0(t)} \sum_{i=0}^{t} (i - \mu_0(t))^2 \times p_i$$

$$\sigma_1^2(t) = \frac{1}{P_1(t)} \sum_{i=t+1}^{L-1} (i - \mu_1(t))^2 \times p_i$$

Class Separability Measure

- Statistical Terms



$$\mu_0(t=3) = \frac{1}{0.6}\sum_{i=0}^{3} i \times p_i$$

$$= \frac{1}{0.6}(0 \times 0.0 + 1 \times 0.2 + 2 \times 0.4 + 3 \times 0)$$

$$= \frac{1.0}{0.6} = 1.67$$

$$\sigma_0^2(t=3) = \frac{1}{0.6}\sum_{i=0}^{t}(i - 1.67)^2 \times p_i$$

$$= \frac{1}{0.6}\left((0-1.67)^2 \times 0.0 + (1-1.67)^2 \times 0.2 + (2-1.67)^2 \times 0.4 + (3-1.67)^2 \times 0\right)$$

$$= \frac{0.13334}{0.6} = 0.22$$

Class Separability Measure

- Observation
  - Pixels in the same class should be with homogeneous intensity.
  - Variance is a good metric for measuring homogeneity.
    - low variance → high homogeneity
    - high variance → low homogeneity
  - Within-Class variance can be applied to evaluate the goodness of a threshold t

Class Separability Measure

- Within-Class Variance $\sigma_w^2(t)$
  - Definition:

$$\sigma_w^2(t) = P_0(t)\sigma_0^2(t) + P_1(t)\sigma_1^2(t)$$

  - σ_w^2-Based Objective Function

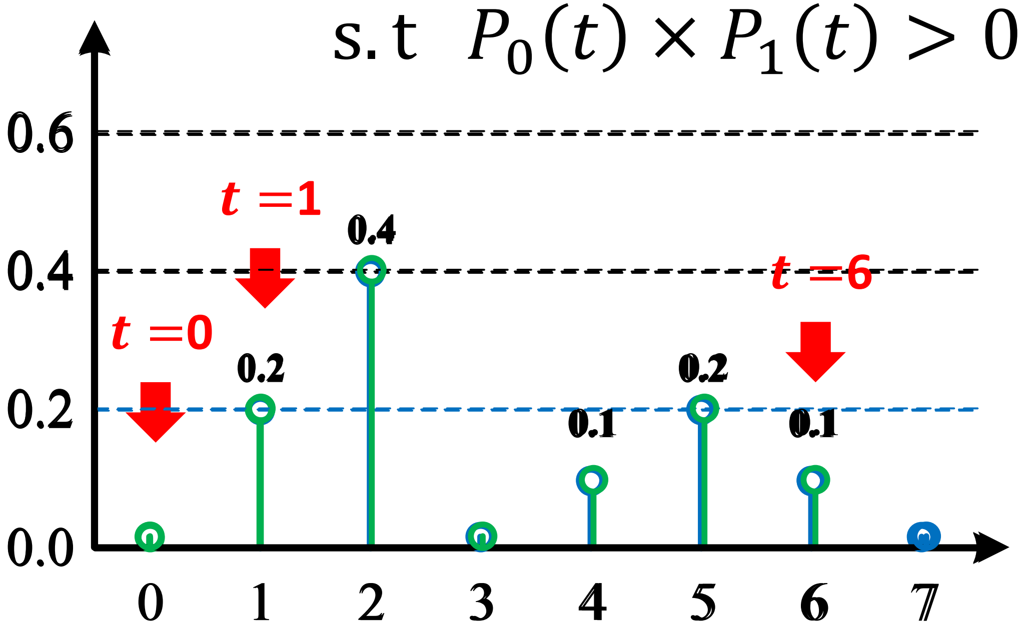
$$t^* = \operatorname{argmin} \sigma_w^2 \qquad \text{s.t} \;\; P_0(t) \times P_1(t) > 0$$

Class Separability Measure

- Within-Class Variance $\sigma_w^2(t)$

$$t^* = \text{argmin}\ P_0(t)\sigma_0^2(t) + P_1(t)\sigma_1^2(t)$$

$$\text{s.t}\ \ P_0(t) \times P_1(t) > 0$$



t^*=2 or 3

t	σ_w^2
$t = 0$	Invalid
$t = 1$	2.50
$t = 2$	0.75
$t = 3$	0.75
$t = 4$	1.16
$t = 5$	2.00
$t = 6$	Invalid

Class Separability Measure

- Between-Class Variance $\sigma_b^2(t)$
  - Definition:

$$\sigma_b^2(t) = P_0(t)P_1(t)\left(\mu_0(t) - \mu_1(t)\right)^2$$

  - σ_w^2-Based Objective Function

$$\sigma_G^2 = \sigma_w^2(t) + \sigma_b^2(t)$$

constant minimize maximize

$\Rightarrow$ $t^* = \operatorname{argmax} \sigma_b^2 \quad \text{s.t} \;\; P_0(t) \times P_1(t) > 0$

Threshold Selection Algorithm

- Initialization
  - initialize four statistical variables

$$P_0 = 0.0 \quad P_1 = 1.0$$

$$Q_0 = 0.0 \quad Q_1 = \sum_{i=0}^{L-1} i \times p_i$$

  - initialize two variables
    - $t^* = 0$: optimal threshold
    - $\sigma_{b,max}^2 = 0$: maximal between-class variance

Threshold Selection Algorithm

- Iteration ($t = 0,1,..,L-1$)
  - Step 1: update four statistical variables in a recursive manner

$$P_0 \leftarrow P_0 + p_t \qquad P_1 \leftarrow P_1 - p_t$$

$$Q_0 \leftarrow Q_0 + t \times p_t \qquad Q_1 \leftarrow Q_1 - t \times p_t$$

  - Step 2: proceed to the next iteration $t \leftarrow t+1$ ***if*** $P_0 = 0.0$ or $P_1 = 0.0$

Threshold Selection Algorithm

- Iteration ($t = 0,1,..,L-1$)
  - Step 3: compute μ_0 and μ_1

$$\mu_0 = \frac{Q_0}{P_0} \qquad \mu_1 = \frac{Q_1}{P_1}$$

  - Step 4: compute between-class variance $\sigma_b^2(t)$

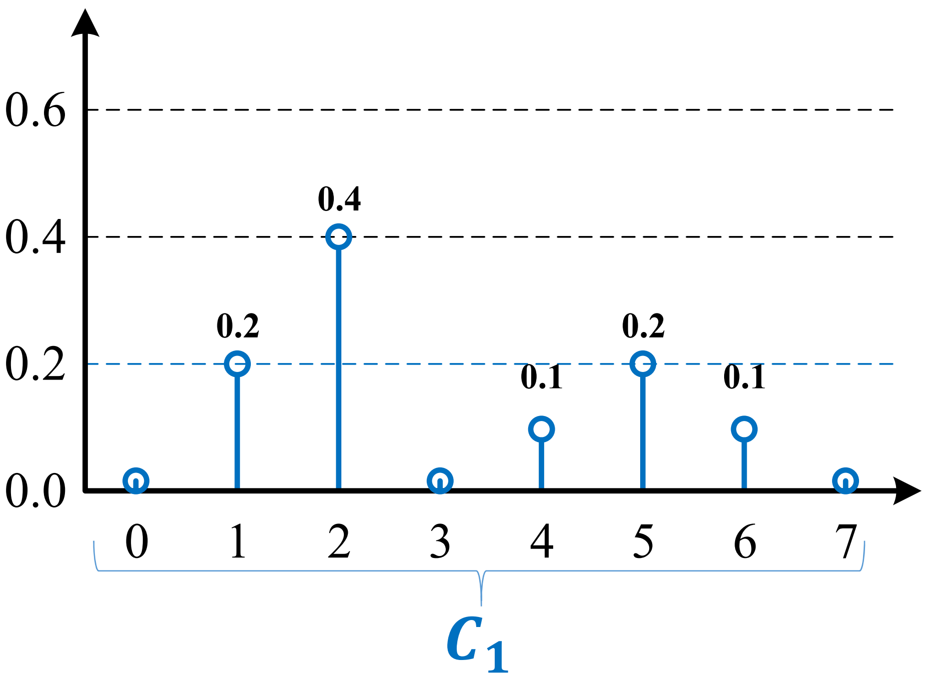
$$\sigma_b^2(t) = P_0 P_1 \times (\mu_0 - \mu_1)^2$$

  - Step 5: update $t^*$and $\sigma_{b,max}^2$ ***if*** $\sigma_b^2(t) \geq \sigma_{b,max}^2$

$$t^* \leftarrow t \qquad \sigma_{b,max}^2 \leftarrow \sigma_b^2(t)$$

Threshold Selection Algorithm

- Example: Initialization Step



$P_0 = 0.0 \qquad P_1 = 1.0$

$Q_0 = 0.0$

$Q_1 = 0 \times 0.0 + 1 \times 0.2 + ..$

$+6 \times 0.1 + 7 \times 0.0$

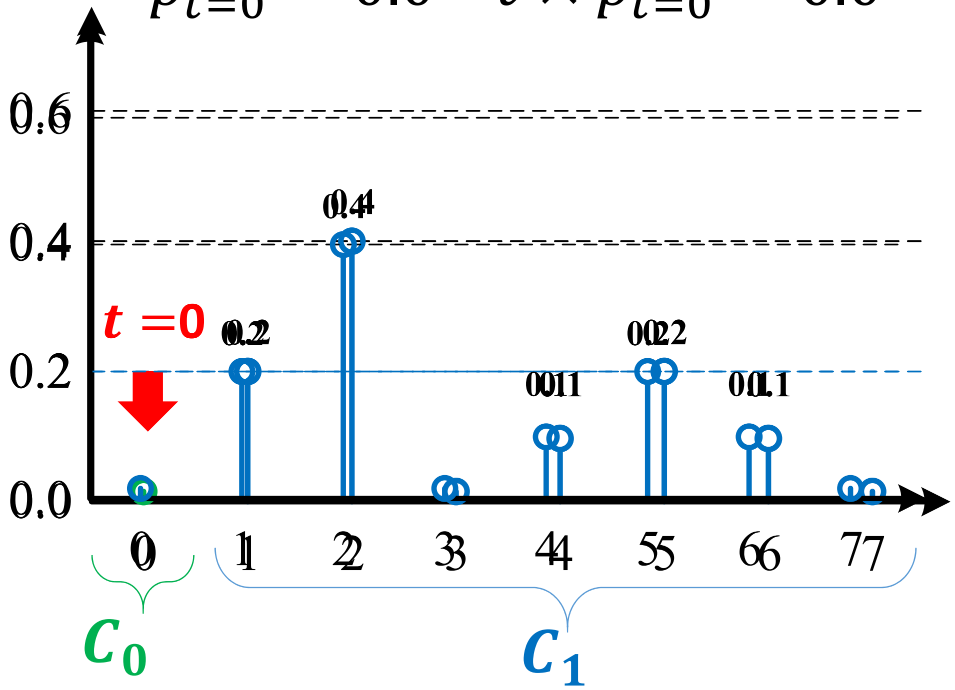
$= 3.0$

$t^* = 0 \qquad \sigma_{b,max}^2 = 0$

Threshold Selection Algorithm

- Example: ($t = 0$)

$p_{t=0} = 0.0 \quad t \times p_{t=0} = 0.0$



Step 1:

$P_0 \leftarrow 0.0 + p_t \Rightarrow P_0 = 0.0$

$P_1 \leftarrow 1.0 - p_t \Rightarrow P_1 = 1.0$

$Q_0 \leftarrow 0.0 + t \times p_t \Rightarrow Q_0 = 0.0$

$Q_1 \leftarrow 3.0 - t \times p_t \Rightarrow Q_1 = 3.0$

Step 2:

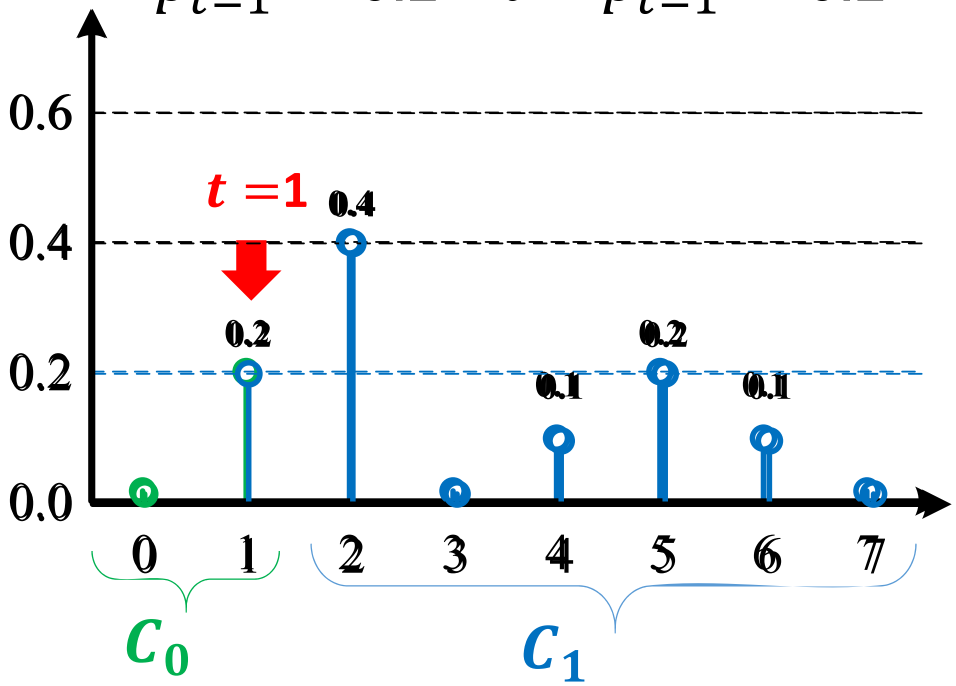
proceed to the next iteration

$t \leftarrow t + 1$ because $P_0 = 0.0$

Threshold Selection Algorithm

- Example:$(t = 1)$

$p_{t=1} = 0.2 \quad t \times p_{t=1} = 0.2$



Step 1

$P_0 \leftarrow 0.0 + p_t \Rightarrow P_0 = 0.2$

$P_1 \leftarrow 1.0 - p_t \Rightarrow P_1 = 0.8$

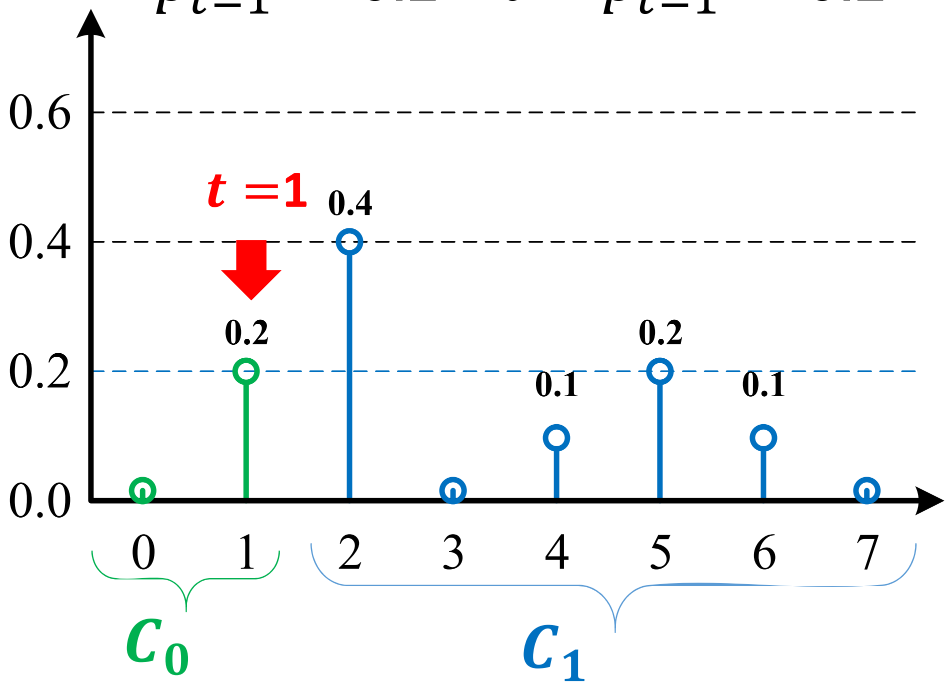
$Q_0 \leftarrow 0.0 + t \times p_t \Rightarrow Q_0 = 0.2$

$Q_1 \leftarrow 3.0 - t \times p_t \Rightarrow Q_1 = 2.8$

Threshold Selection Algorithm

- Example:$(t = 1)$

$p_{t=1} = 0.2 \quad t \times p_{t=1} = 0.2$



Step 3

$\mu_0 = \frac{0.2}{0.2} = 1.0 \quad \mu_1 = \frac{2.8}{0.8} = 3.5$

Step 4

$$\sigma_b^2(t) = 0.2 \times 0.8 \times (1.0 - 3.5)^2 = 1.0$$

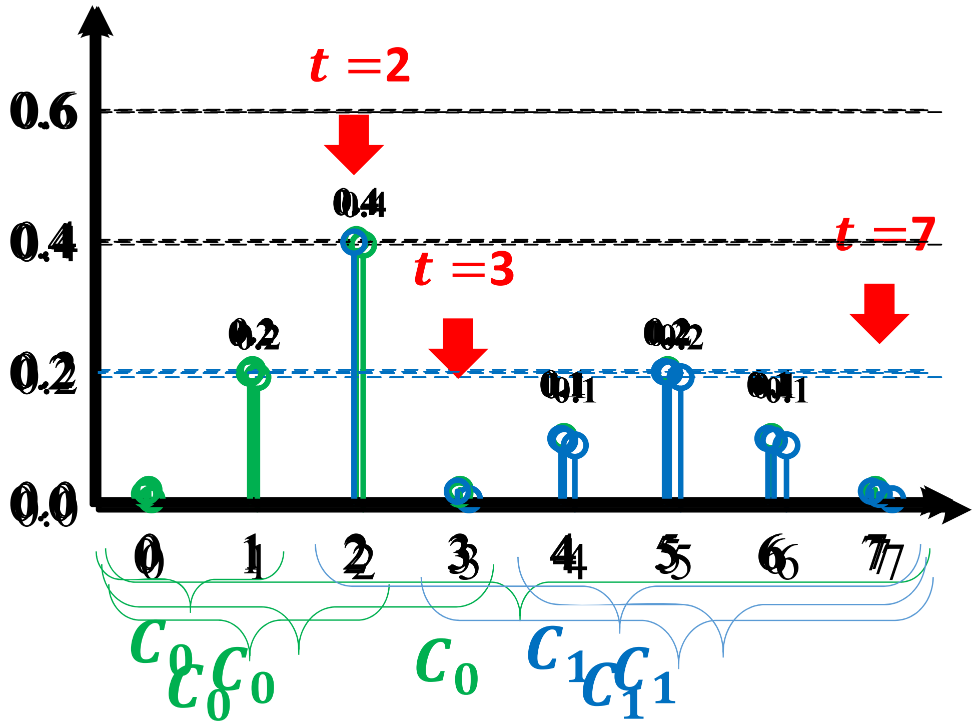
Step 5

$t^* \leftarrow 1 \qquad \sigma_{b,max}^2 \leftarrow 1.0$

Threshold Selection Algorithm

- Example

$t^* = 3$ $\sigma^2_{b,max} = 2.66$



2	2	2	1	4
2	1	6	4	1
2	2	1	6	5
2	2	5	5	5

Thresholding
$t^* = 3$

0	0	0	0	1
0	0	1	1	0
0	0	0	1	1
0	0	1	1	1


thank you
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